Beer consumption in UK per year

$Q = f(m, p, r, s)$ where

$$f(m,p,r,s) = (1.058)\, m^{0.136}\, p^{-0.727}\, r^{0.914}\, s^{0.816}$$

m: aggregate real income

p: average price of beer

r: average price of all other consumer goods and services

s: strength of beer

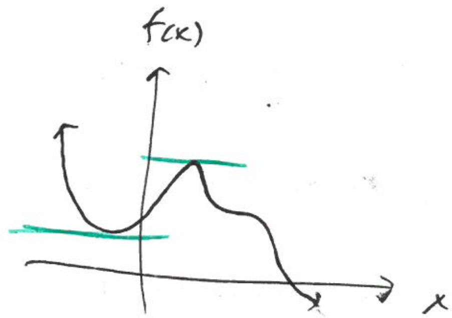
which partials are $+$ and $-$

and what is your interpretation?

7.3 Maxima and minima

In Calc 131 to find minima & maxima

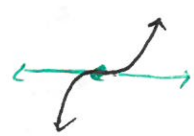
set $f'(x) = 0$ to find where tangent line has flat slope



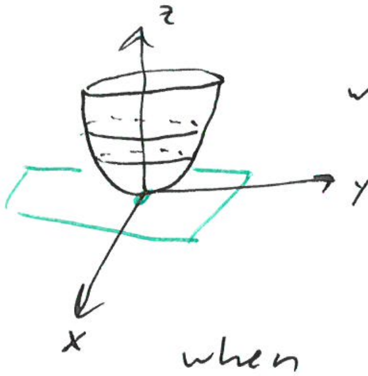
Call x where $f'(x) = 0$ critical points.

Critical points can give a max  $y = -x^2$

min  $y = x^2$

inflection point  $y = x^3$

How do we find minima and maxima in 2 variables?



want the tangent plane to be flat

when* does this happen?

when $f_x = 0$ and $f_y = 0$

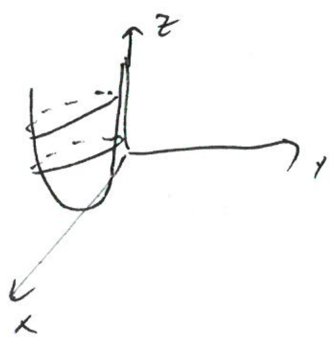
- A critical point (a, b) for the function $f(x,y)$ occurs when $f_x(a,b) = 0$ <u>and</u> $f_y(a,b) = 0$.

- If a function $f(x,y)$ has a minimum or maximum at (a,b) then (a,b) is a critical point.

ex

$$f(x,y) = 2x^2 - 2xy + 5y^2 - 6x + 5$$

has a minimum point. Find its coordinates.



$$\frac{\partial f}{\partial x} = 4x - 2y - 6$$

$$\frac{\partial f}{\partial y} = -2x + 10y$$

so

$$\begin{cases} 4x - 2y - 6 = 0 \\ -2x + 10y = 0 \end{cases}$$

system of equations

$$\begin{cases} 4x - 2y = 6 \\ 2x = 10y \end{cases} \Rightarrow \begin{cases} 4x - 2y = 6 \\ x = 5y \end{cases}$$

substitute $x = 5y$ to 1st eq.

$$4(5y) - 2y = 6$$

$$\Rightarrow 20y - 2y = 6$$

$$\Rightarrow 18y = 6$$

$$\boxed{y = \tfrac{1}{3}}$$

since $x = 5y$

$$\boxed{x = \tfrac{5}{3}}$$

minimum at $\left(\tfrac{5}{3}, \tfrac{1}{3}\right)$

what is the value?

$$f\left(\tfrac{5}{3}, \tfrac{1}{3}\right) = 2\left(\tfrac{5}{3}\right)^2 - 2\left(\tfrac{5}{3}\right)\left(\tfrac{1}{3}\right) + 5\left(\tfrac{1}{3}\right)^2 - 6\left(\tfrac{5}{3}\right) + 5 = 0$$

ex/ Find the points where $f(x,y)$ has a possible max or min:

$$f(x,y) = -3x^2 + 7xy - 4y^2 + x + y$$

$$f_x = -6x + 7y + 1$$

$$f_y = 7x - 8y + 1$$

$$\begin{cases} 0 = -6x + 7y + 1 \\ 0 = 7x - 8y + 1 \end{cases}$$

$$\begin{cases} x = \frac{7}{6}y + \frac{1}{6} \\ 0 = 7x - 8y + 1 \end{cases}$$

plug in

$$0 = 7\left(\frac{7}{6}y + \frac{1}{6}\right) - 8y + 1$$

$$0 = \frac{49}{6}y + \frac{7}{6} - 8y + 1$$

$$0 = \frac{49}{6}y - \frac{48}{6}y + \frac{7}{6} + \frac{6}{6}$$

$$0 = \frac{1}{6}y + \frac{13}{6}$$

$$0 = y + 13$$

$$\boxed{y = -13}$$

plug in

$$x = \frac{7}{6}y + \frac{1}{6}$$

$$x = \frac{7}{6}(-13) + \frac{1}{6}$$

$$= \frac{-91}{6} + \frac{1}{6}$$

$$= \frac{-90}{6}$$

$$\boxed{x = -15}$$

critical point at $(-15, -13)$

But how do we know if its a max, min, or neither?

2nd Derivative test

For 1 variable functions the 2nd derivative tells us about concavity.

A similar test exists for 2 variables

Let

$$D(x,y) = \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2$$

$$= f_{xx} \cdot f_{yy} - f_{xy}^{\ 2}$$

If (a,b) is a critical point

1.) If

$$D(a,b) > 0 \quad \underline{\text{and}} \quad \frac{\partial^2 f}{\partial x^2}(a,b) > 0$$

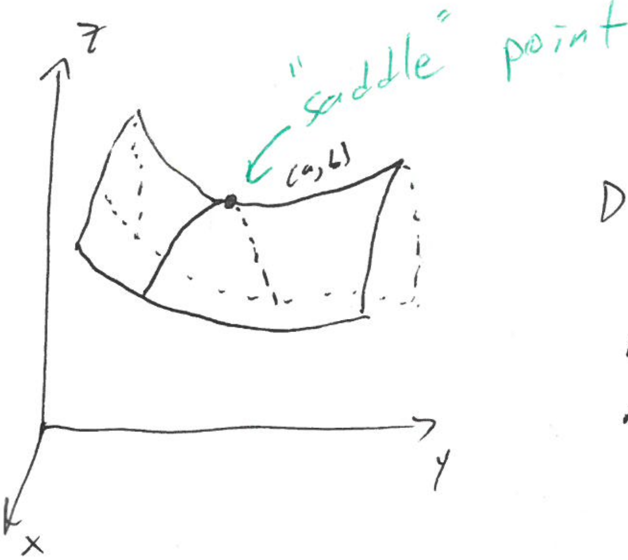
then $f(x,y)$ has a relative <u>minimum</u> at (a,b)

2.) If $D(a,b) > 0$ <u>and</u> $\frac{\partial^2 f}{\partial x^2}(a,b) < 0$

then $f(x,y)$ has a relative <u>maximum</u> at (a,b)

3.) If $D(a,b) < 0$ then $f(x,y)$ is neither a maximum or minimum at (a,b)

4.) If $D(a,b) = 0$ then test inconclusive.



$D(a,b) < 0$

relative maximum in x direction

relative minimum in y direction

back to earlier example

$$f(x,y) = -3x^2 + 7xy - 4y^2 + x + y$$

has a critical point at $(-15,-13)$

but what is it?

$$f_x = -6x + 7y + 1$$

$$f_y = 7x - 8y + 1$$

$$f_{xx} = -6$$

$$f_{yy} = -8$$

$$f_{xy} = 7$$

$$D(x,y) = (-6)(-8) - 7^2$$

$$= 48 - 49 = -1$$

$D(-15,-13) < 0$ so

$(-15,-13)$ is ~~max~~

a saddle point.